CHAPTER 2.1

Extending Bohr: Sommerfeld's early atomic theory, 1913-1916

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Abstract

The development of the so-called "Bohr-Sommerfeld theory" has been the subject of considerable scrutiny.[1] Most historical studies of Sommerfeld's contributions have aimed at a reconstruction of the theoretical prerequisites that sparked the subsequent development towards quantum mechanics. The aim of this paper is different: I do not portray Sommerfeld's achievement with a focus on the crucial innovations for the ensuing development of quantum theory, but from the perspective of Sommerfeld's contemporary correspondence[2] – the main source for such a historical reconstruction – and his biography.[3] The focus is on the dynamics that drove Sommerfeld's atomic research in the time span between the publication of Bohr's model (July 1913) and the presentation of Sommerfeld's memoirs to the Bavarian Academy of Science (December 1915).[4]

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1. Jammer (1966); Nisio (1973); Mehra and Rechenberg (1982); Kragh (1985); Robotti (1986); Kragh (2012).

2. Sommerfeld (2000).

3. Eckert (2013a).

4. Sommerfeld (1915a,1915b). For a historical discussion of these treatises see Eckert (2013b, 2014).

Key words: Niels Bohr; Arnold Sommerfeld; atomic theory; Zeeman effect; Stark effect.

1. The first reaction to Bohr's model

Sommerfeld reacted swiftly to Bohr's model, in fact earlier than any other physicist outside Rutherford's circle: "The problem of expressing the Rydberg-Ritz constant by Planck's h has for a long time been in my mind," Sommerfeld wrote to Bohr about the "great feat" of the model. "Will you also apply your atom model to the Zeeman effect? I intended to deal with the latter."[5] The Zeeman effect, i. e., the splitting of spectral lines in a magnetic field, was the subject of Sommerfeld's most recent research effort (see below). Planck's h had been in the focus of Sommerfeld's agenda for a couple of years. In his "h-hypothesis," presented at the First Solvay Congress in 1911,[6] Sommerfeld had assumed that the energy E and time τ involved in "molecular elementary processes" are related via $E\,\tau = h$. In the photoelectric effect, for example, this hypothesis implied that an atom accumulates radiation within a time span τ until an electron is emitted with energy E. Sommerfeld applied this hypothesis also to the production of x-rays (Bremsstrahlen), radioactive decay and other elementary processes. But it was doomed to failure. The accumulation time in the photoelectric effect, for example, could amount to years, as Sommerfeld admitted in July 1913.[7] Another quantum topic on Sommerfeld's agenda concerned the kinetic theory of gases. Sommerfeld presented some ideas on gas quantization following Debye's approach in the theory of the specific heat of solids in April 1913 at a conference in Göttingen ("gas week"). But he could not bring this theory "to a satisfying completion".[8]

5. Sommerfeld to Bohr, 4 September 1913. NBA. English translation in Bohr (1981), p. 123. The original German text is reproduced on p. 603 and in Sommerfeld (2000), p. 477.

6. Sommerfeld (1911). For more detail on the h-hypothesis see Hermann (1969), pp. 125-126, and Eckert (2011).

7. Sommerfeld (1913b), p. 711. See also Wheaton (1983), pp. 180-189.

8. Sommerfeld to Hilbert, 14 October 1913. SUB, Cod. Ms. D. Hilbert 379 A.

While these attempts to understand "Planck's *h*" were rapidly losing ground, the other item mentioned in Sommerfeld's first reaction, the Zeeman effect, received new attention in 1912 with the discovery of the Paschen-Back effect, i.e., the transformation of the "anomalous" line splitting in weak magnetic fields into a "normal" line splitting in strong magnetic fields. Sommerfeld attempted to account for this phenomenon by extending Lorentz's classical theory of the Zeeman effect. He replaced the isotropic oscillation of an electron in Lorentz's model by anisotropic oscillations (with three slightly different oscillation frequencies along the three directions of a Cartesian coordinate system).[9] "These days I have conceived a paper on the Zeeman phenomenon following Paschen-Back," he wrote in January 1913 to Carl Runge, an expert on spectroscopy, "and I would like to know from you whether it is new."[10] He also was in close contact with Friedrich Paschen who shared with him his recent measurements on the Zeeman effect.[11]

Another correspondent with whom Sommerfeld exchanged his ideas on the Zeeman and Paschen-Back effect was Woldemar Voigt, an authority on magneto-optics. Voigt had raised the idea of anisotropically bound electrons much earlier.[12] The mutual exchange almost turned into a "very unpleasant" rivalry.[13] Sommerfeld avoided a clash by acknowledging Voigt's primacy, but he expressed doubt about Voigt's trust in the underlying physical model. "As long as we do not have a theory of spectral lines each theory of magneto-optics remains piecemeal."[14]

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9. Sommerfeld (1913a).

10. Sommerfeld to Runge, 17 January 1913. DMA, HS 1976-31. Also in Sommerfeld (2000), pp. 468-469.

11. Paschen to Sommerfeld, undated (ca. 10 March 1913), 18 March 1913, 21 March 1913, and 1 April 1913. DMA, HS 1977-28/A, 253. The first of these letters is also in Sommerfeld (2000), pp. 469-471.

12. Voigt to Sommerfeld, 26 January 1913. DMA, HS 1977-28/A,347.

13. Sommerfeld to his wife, 14 March 1913. Private estate.

14. Sommerfeld to Voigt, 24 March 1913. DMA, NL 89, 015. Also in Sommerfeld (2000), pp. 471-474.

2. Proving grounds

Bohr's model offered the prospect for such a theory of spectral lines. The most persuasive evidence concerned a series of spectral lines that had been named "Pickering lines" and ascribed to hydrogen.[15] According to Bohr's theory these lines belonged to the spectrum of ionized helium.[16] Although the issue was not yet definitively settled in the spring of 1914, Bohr's arguments were seriously considered by the leading spectroscopists.

With regard to the Zeeman effect, however, Bohr's model had little to offer. When Sommerfeld addressed the Zeeman effect again in a paper in 1914, he did not use Bohr's model but Voigt's theory.[17] Bohr regarded the Zeeman effect merely as "promising on account of the close analogy between the hypothesis of the universal constancy of the angular momentum of the electrons and the theory of magnetons."[18] The question of an elementary unit of the magnetic moment had been subject of previous debates. But Bohr's subsequent note on the Zeeman effect did not live up to these expectations.[19] In a letter to Paul Langevin, dated 1 June 1914, Sommerfeld expressed the opinion that there is "much truth in Bohr's model; and yet I believe that it has to be re-interpreted in a fundamental manner. At the moment I find it particularly disturbing that it yields a wrong value for the magneton."[20] Bohr himself was well aware of this discrepancy, as he noted in an unpublished draft.[21]

Another proving ground for Bohr's model was the splitting of

15. Edward Charles Pickering taught physics at Massachusetts Institute of Technology and investigated stellar spectra at Harvard College Observatory.

16. Kragh (2012), pp. 69-71.

17. Sommerfeld (1914).

18. Bohr to Sommerfeld, 23 October 1913. DMA, HS 1977-28/A,28. Also in Sommerfeld (2000), pp. 478-479.

19. Bohr (1981), p. 325. Bohr's note was published in March 1914.

20. Sommerfeld to Langevin, 1 June 1914. ESPC, Langevin papers, L 76/53. Also in Sommerfeld (2000), pp. 484-485.

21. Bohr (1981), p. 261. The experimentally determined atomic unit for the magnetic moment ("Weiss magneton") was an order of magnitude smaller than the magnetic moment due to an electron rotating in the smallest circular orbit around the hydrogen nucleus ("Bohr's magneton").

hydrogen spectral lines in electric fields discovered in 1913 by Johannes Stark and, independently, by Antonio Lo Surdo.[22] The new phenomenon was immediately perceived as a challenge. Emil Warburg, the president of the Physikalisch-Technische Reichsanstalt (PTR) in Berlin, regarded the Stark effect as a phenomenon "which cannot be explained on the ground of classical electrodynamics." He regarded Bohr's theory as promising but concluded that it required some modifications.[23] The astronomer Karl Schwarzschild approached the problem with an analogy from celestial mechanics: the case of a planet orbiting around two suns. The configuration of the Stark effect corresponded to the case when one sun (nucleus) was moved to infinity by increasing at the same time its mass (charge) so that its gravity (electric field) was perceived by the planet (electron) as a homogeneous superposition to the central field of the first sun (nucleus). The periods of the distorted electron orbits in this configuration, however, did not agree with Stark's experimental observations.[24] In January and February 1914, Bohr exchanged his own ideas on the Stark effect with Warburg and Schwarzschild. He assumed that the electric field deforms the circular into elliptical orbits. In agreement with experiments he obtained a frequency shift proportional to the strength of the applied electric field, but the magnitude of the frequency shift was about 30 percent too high.[25]

A third proving ground for Bohr's theory opened up when James Franck and Gustav Hertz measured the "ionization potential" of atoms in the passage of cathode rays through a mercury vapor. Bohr interpreted these experiments differently: the observed bumps did not correspond to ionization but to the energy differences between the stationary states in the mercury atoms.[26]

All of these proving grounds for Bohr's model were closely observed and discussed in Sommerfeld's colloquium during the sum-

22. Leone et al. (2004).

23. Warburg (1913), p. 1259 and p. 1266.

24. Schwarzschild (1914a).

25. Bohr (1981), pp. 321-323.

26. Kragh (2012), pp. 143-146.

mer semester of 1914. By the end of this semester Bohr visited Sommerfeld and offered the participants of his colloquium a first-hand presentation "On Bohr's atomic model, with particular emphasis on the spectra of helium and hydrogen."[27] Thus atomic theory ranked high on Sommerfeld's research agenda. Yet none of these proving grounds yielded irrevocable corroboration of Bohr's theory. Sommerfeld did not yet feel compelled to turn his wait-and-see attitude into a crash program on Bohr's model.

3. A lecture course on "Zeeman effect and spectral lines"

The First World War delayed research in atomic theory further. Sommerfeld, at the age of 45, was not sure whether he would be drafted for military service. "Judging from what I have heard at the general headquarters, it seems that they are not very eager to make use of my services," he wrote to Schwarzschild in October 1914. "If they leave me at home, it's just as well since I've never felt myself militarily strong." He asked Schwarzschild for news about the theory of the Zeeman effect because he intended to dedicate a course of lectures in the coming winter semester on "Zeeman effect and spectral lines" if he would not have to serve for the military.[28]

When it became clear that he would not be drafted, Sommerfeld resumed his effort towards a theory of the Zeeman effect and dedicated his special lecture course to this subject–as foreseen in his correspondence with Schwarzschild.[29] Another correspondent on this topic was Paschen. "You will be in for a bad shock when you see the complicated Zeeman types," Paschen informed him about new measurements on the anomalous Zeeman effect.[30] In February 1915 the shoptalk between the Tübingen spectroscopist and the Munich

27. Physikalisches Mittwoch-Colloquium. DMA, 1997-5115. Bohr's presentation on 15 July 1914 was titled "Über das Bohrsche Atommodell, insbesondere die Spektren von Helium und Wasserstoff."

28. Sommerfeld to Schwarzschild, 31 October 1914. SUB, Schwarzschild 743. Also in Sommerfeld (2000), pp. 485-487.

29. Sommerfeld to Schwarzschild, 18 and 30 November 1914. SUB, Schwarzschild 743. Voigt (1913); Schwarzschild (1914b); Sommerfeld (1914).

30. Paschen to Sommerfeld, 15 December 1914. DMA, HS 1977-28/A,253.

theorist turned to the "Pickering series". "Due to Bohr's theory Fowler suspects that these are helium lines," Paschen reported about the opinion of the leading British spectroscopist, but he wished to perform the pertinent measurements himself. "We will publish the evidence only when the experiments are completed. Unfortunately this cannot happen now because of the war."[31] The dispute about the "Pickering lines" had already surfaced in Bohr's Munich colloquium talk ("with particular emphasis on the spectra of helium and hydrogen"). Sommerfeld must have discussed this issue again in his lectures, so that Bohr's theory became again a hot subject towards the end of the winter semester 1914/15. "I have lectured on Bohr during this semester and am extremely interested in his theory as far as the war permits," Sommerfeld wrote to Wilhelm Wien. "Today's 100,000 Russians, however, are even more beautiful than Bohr's explanation of the Balmer series. I have marvelous new results in this regard."[32]

From the available archival sources it is not clear what "new results" Sommerfeld had obtained in February 1915. They must have concerned the Stark effect, because his assistant Wilhelm Lenz, who spent his military service at the western front in northern France, congratulated Sommerfeld in April 1915: "I got excited about your discovery with regard to the Bohr model and the Stark effect, and I am very curious about the further progress."[33] Sommerfeld also mentioned his "discovery" in another letter to Wilhelm Wien: "During the past semester I obtained an interesting approach for the Stark effect from Bohr's theory of the hydrogen lines."[34] The crucial idea for Sommerfeld's "discovery" originated from a paper which Stark had recently published on the "fine-decomposition" of the Balmer series in an electric field: the number of components in-

31. Paschen to Sommerfeld, 7 February 1915. HS 1977-28/A,253.

32. Sommerfeld to W. Wien, 22 February 1915. DMA, NL 56, 005, C III. Also in Sommerfeld (2000), pp. 491-493. The remark about the "100,000 Russians" alluded to the defeat of the 10th Russian army in the "winter battle" in East Prussia when about 100,000 Russian soldiers became prisoners of war.

33. Lenz to Sommerfeld, April 1915. DMA, NL 89, 059.

34. Sommerfeld to W. Wien, 3 May 1915. DMA, NL 56, 005. Also in Sommerfeld (2000), pp. 493-494.

creased with the index of the Balmer line.[35] "The number of line decompositions in the Stark effect of hydrogen" had also been discussed on 16 January 1915 in Sommerfeld's colloquium.[36] If a Balmer line consists of coinciding lines, originating from different orbits with equal energies, then the coincidence is removed when a disturbance such as an applied electric field deforms these orbits so that they have no longer equal energies. This must have been the idea that prompted the "interesting approach" with which Sommerfeld now intended to extend Bohr's model.

4. Two papers for the Bavarian Academy of Science

In the same letter to Wien in which Sommerfeld mentioned his "discovery" he also wrote that he had no time to elaborate upon it because "problems of war physics" had cropped up.[37] Einstein's new theory of general relativity further added to the distraction. "I lectured this semester on relativity as presented by Einstein in his recent Berlin communication and am enthusiastic about it, almost as much as about Bohr in the preceding semester," Sommerfeld wrote to Schwarzschild at the end of the summer semester 1915.[38] He would certainly have mentioned the extension of Bohr's model if he had elaborated upon it already by this time.

The event that finally triggered this elaboration was presumably Bohr's paper "On the Quantum Theory of Radiation and the Structure of the Atom," published in September 1915.[39] "Recent work by Bohr" was the theme of Sommerfeld's presentation in his colloquium on 27 November 1915.[40] A few days earlier Paschen had informed him about his measurements of "Bohr's H and He series"

35. Stark (1914).

36. Physikalisches Mittwoch-Colloquium. DMA, 1997-5115.

37. Sommerfeld to W. Wien, 3 May 1915. DMA, NL 56, 005. Also in Sommerfeld (2000), pp. 493-494.

38. Sommerfeld to Schwarzschild, 31 July 1915. SUB, Schwarzschild 743. Also in Sommerfeld (2000), pp. 498-499.

39. Bohr (1981), p. 335. Bohr's paper is reprinted on pp. 392-413. On Kossel's contributions see Heilbron (1967).

40. Physikalisches Mittwoch-Colloquium. DMA, 1997-5115.

and concluded that "Bohr's theory is exactly confirmed except the complicated structure of the lines 4686 etc."[41] Bohr's theory correctly ascribed "4686" to the spectrum of ionized helium, but could not explain the remaining discrepancy: its fine structure. At this point Sommerfeld's earlier "discovery" about the fine-decomposition of lines in the Stark effect offered a solution. By November 1915 he must have drafted his ideas in the form of manuscripts and sent them to Paschen and Einstein, because both mentioned these manuscripts in their response.[42] "So the 'discrepancy' is theoretically required!", Paschen congratulated. "There is nothing like a nice theory!"[43]

On 6 December 1915, Sommerfeld presented the first of these two papers to the Bavarian Academy of Science. In one paragraph he revealed that his effort on the Zeeman effect had nurtured his view that Bohr's model requires a basic extention, and that he received from the Stark effect the idea how this could be accomplished: "Lorentz's fundamental theory of the Zeeman effect is based on the assumption that in each spectral line three equal fundamental oscillations of a quasi-elastic and isotropically vibrating electron coincide. The magnetic field does not create new oscillations, but decomposes the original ones ... . This view may be transferred immediately to the Stark effect of the Balmer series. According to our view there is a number of frequencies of different origin coinciding in each Balmer line. The electric field affects the various elliptic orbits in a different manner and thus decomposes the originally coinciding frequencies."[44] Three days later, on 9 December 1915, Einstein informed Sommerfeld that "Planck works on a similar problem like you (quantization of the phase space of molecular

41. Paschen to Sommerfeld, 24 November 1915. DMA, HS 1977-28/A,253. Also in Sommerfeld (2000), pp. 499-500.

42. Einstein to Sommerfeld, 28 November 1915. DMA, HS 1977-28/A,78. Also in Sommerfeld (2000), pp. 500-503. Paschen to Sommerfeld, 12 December 1915. DMA, HS 1977-28/A,253. Also in Sommerfeld (2000), pp. 504-506.

43. Paschen to Sommerfeld, 30 December 1915. DMA, HS 1977-28/A,253. Also in Sommerfeld (2000), pp. 513-514.

44. Sommerfeld (1915a), p. 449.

systems)."[45] On 22 December 1915 Kossel presented in Sommerfeld's colloquium the results of a dissertation by a student of Manne Siegbahn which contained conclusive evidence that Kossel's interpretation of x-ray spectra in terms of Bohr's theory was correct.[46] From this observation Sommerfeld concluded that the same mechanism, the decomposition of originally coinciding frequencies, is also the cause for the previously unexplained x-ray doublets.

In his second memoir, presented to the Bavarian Academy after the Christmas holidays on 8 January 1916, Sommerfeld showed that a decomposition of lines not only results from an external disturbance (like the electric field in the Stark effect) but also from the relativistic motion of an electron in elliptic orbits with quantized excentricities.[47] The fine-structure of spectral lines was thus explained as a relativistic effect, and the doublets in the x-ray spectra appeared as an extension of the fine structure in the hydrogen spectrum. "I show that for all elements from $Z = 20$ to $Z = 60$, where measurements are available, $\Delta \nu/(Z-1)^4 = \Delta \nu_H$! $\Delta \nu$ = difference of frequencies of the x-ray doublets, $\Delta \nu_H$ = difference of frequencies of the hydrogen doublet," Sommerfeld wrote to Schwarzschild. The hydrogen doublets appear "extremely magnified" as x-ray doublets in the spectra of heavy elements. "After all I am convinced that my theory of the quantized ellipses correctly accounts for the physical facts, and definitively unveils the riddle of the spectral lines."[48]

Despite such success Sommerfeld regarded the theory as preliminary. "My spectral lines are finally printed in the Academy in a provisional manner," he explained to Wilhelm Wien, the editor of the *Annalen der Physik*. "They will appear in the *Annalen* in a refined form."[49]

45. Einstein to Sommerfeld, 9 December 1915. DMA, HS 1977-28/A,78. Also in Sommerfeld (2000), pp. 503-504. On Planck's work, see Eckert (2010).

46. Physikalisches Mittwoch-Colloquium. DMA, 1997-5115. See also Heilbron (1967), p. 465.

47. Sommerfeld (1915b).

48. Sommerfeld to Schwarzschild, 28 December 1915. SUB, Schwarzschild. Also in Sommerfeld (2000), pp. 509-511.

49. Sommerfeld to W. Wien, 10 February 1916. DMA, NL 56, 010. Also in Sommerfeld (2000), pp. 525-526. The stage that led to the *Annalen* paper is beyond the scope of this article. See Sommerfeld (2000), pp. 436-445, 514-566.

5. Conclusion

Sommerfeld's extension was not a single feat but a process that went through several stages. The stage considered in this paper, from Bohr's "trilogy" in summer 1913 to Sommerfeld's treatises for the Bavarian Academy in December 1915 and January 1916, marked the beginning of what became known as the Bohr-Sommerfeld theory. Initially Sommerfeld aimed for a theory of the Zeeman effect (including the Paschen-Back effect) and the Stark effect. Bohr's theory failed in this respect. In the course of lectures in the winter semester 1914/15, when Sommerfeld reviewed recent spectroscopic research, he received from a recent paper on the Stark effect the idea how to extend Bohr's theory so that it also applies to the splitting of lines in magnetic and electric fields. The main actors with whom Sommerfeld corresponded on these issues were Schwarzschild and Paschen. The local fora for discussing pertinent results and introducing new concepts prior to publication were his special lecture and his colloquium.

It is ironic that Sommerfeld could not transform the idea which he received from Stark's paper on the decomposition of Balmer lines in electric fields into a theory of the Stark effect itself. This was left to Schwarzschild and Paul Epstein for a dramatic rivalry in spring 1916. Nor was the theory of the Zeeman effect part of Sommerfeld's extension. He reserved this for a separate study in summer 1916, but could only recover the normal Zeeman effect. The anomalous Zeeman effect remained a bone of contention for the Bohr-Sommerfeld model. It climaxed in 1921 when Sommerfeld reinterpreted Voigt's theory in terms of quantum theory and offered it to his student Werner Heisenberg (then in his third semester) for further elaboration.[50] But this is another story.

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50. Cassidy (1979).

ARCHIVAL SOURCES

DMA Deutsches Museum, Archiv. Munich.
ESPC Ecole Supérieure de Physique et de Chimie industrielles de la ville de Paris, Centre de ressources historiques, Paris.
SUB Staats- und Universitäsbibliothek, Göttingen.

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